PROBABILITY DISTRIBUTION OF DEBRIS FLOW DEPTH
AND ITS IMPLICATION IN RISK ASSESSMENT

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ABSTRACT

Debris flow moves in manner of successive surges and deposits by piling of surges. The surge occurs randomly and varies in properties and magnitude. This study explores the probability distribution of velocity and derives the distribution of flow depth based on observations in Jiangjia Gully in the southwest of China. The Weibull distribution is found to be well applicable to both the velocity and depth, with parameters varying in a rather small range. Therefore, the distribution is expected to hold in general for debris flows in different conditions and can be used to estimate the discharge of a potential debris flow. The estimated quantity is better than those inferred from the rainfall at a given frequency because it incorporates both the variation of surges and the real condition of the valley. In conclusion, The distribution provides a more reliable method of risk assessment of debris flow.

Key words: debris flow; surge; Weibull distribution; discharge estimation; risk assessment

INTRODUCTION

Debris flow of high density moves as surge wave (e.g., Blackwelder, 1928; Sharp & Nobles, 1953; Pierson, 1980, 1986; Takahashi, 1991; Iverson, 1997; Major, 1997; Suárez et alii, 2005; Liu et alii, 2008, 2009); it leaves deposit retaining configurations of the surge, such as the lobate front and layer, lateral levee, inverse grading, and blunt margins (Naylor, 1980; Costa & Jarrett, 1981; Sohn, 2000; Sharp & Nobles, 1953; Johnson, 1970; Costa, 1984; Whipple & Dunne, 1992; DeGraff, 1994). As the living surge is hardly seen in the field, the deposit proves the “fossil” for deriving parameters of debris flow as fluid of visco-plasticity (Middleton & Hampton, 1973, 1976; Lowe, 1975, 1976, 1982; Coussot et alii, 1996).

Deposit is a focus in debris flow studies since most disasters are caused by surge impulsion and inundation. In the previous studies, the deposition spread and run-out distance are estimated in various ways (Bathurst, et alii, 1997; Schilling & Iverson, 1997; Takahashi & Yoshida, 1979; Hulme, 1974; Harvey, 1984; Mizuyama & Uehara, 1983; Liu & Tang, 1995). However, these methods are based either on empirical relationships or simplified dynamics largely ignoring the varieties within surges. They make unique and certain prediction for a potential event under given environment. But observations have shown that the deposit is formed by aggregations of successive surges that vary considerably in many ways (Major, 1997; Vallance & Scott, 1997; Sohn et alii, 1999). Then a debris flow involves a stochastic process that cannot be determined by the environment conditions.

Fortunately, Jiangjia Gully (JJG) in the southwest China exhibits a variety of debris flow appearances and allows real-time and systemic observation (Li et alii, 1983; Liu et alii, 2008, 2009). This paper tries to explore the deposit features by using observation data in the last fifty years and find the probability distribution of deposit depth.
where \( P (v) \) is the probability of surge with velocity bigger than \( v \). Statistics for some events are listed in Table 1.

As for the flow depth, it is found to be related to velocity by a power law (Li et alii, 2005; 2010):

\[
v = k h^n
\]

This holds on average for all events of debris flow in JJG. The coefficient \( k \) varies with channel condition (e.g. the roughness). Besides, \( k \) varies little around 6.0 and the exponent \( n \) is about 0.40 on average (Li et alii, 2005). Accordingly, the flow depth, as a power function of velocity, also satisfies the Weibull distribution (Fig.3):

\[
P (h) = C \exp (- \frac{h}{a_H}^b)
\]

with the parameter \( a_H = k a \), and \( b_H = b n \). The parameters can be estimated roughly, for example, by the average \( a (6.42) \) and \( b (4.11) \) in table 1 together with the statistic results of \( k \) and \( n \) in Eq.(3). An estimate on average is

\[
a_H = (k a)^b \sim (6/6.42)^{1.1} \sim 0.80,
\]

\[
b_H = bn \sim 4.11 \times 0.4 \sim 1.60
\]

**FIELD OBSERVATION**

Debris flow in JJG occurs at high frequency and in a variety of appearances. Observation has continued since 1960s and a huge dataset is available now for systemic analysis (for more information of JJG, see, e.g., Li et alii, 1983; Davies, 1990; Li et alii, 2003, 2004; Liu et alii, 2009). Each debris flow contains dozens or even hundreds of surges and the deposit of a single surge looks like a “frozen” surge and keeps the same configuration. The photo in Fig.1 clearly shows the flowing surges and the deposited surges on the gentle slope outside the channel. There is a remarkable similarity between surges in motion (bright in the center) and in termination (black and grey) (Fig. 1), which acts as the unit of deposition. Assumed as a Bingham fluid, a surge deposits when the shear stress is smaller than the yield strength:

\[
\tau < \rho g j h
\]

where \( \tau \) is the shear stress, \( \rho \) the density of flow, \( g \) the gravity acceleration, \( j \) the slope gradient of the channel, and \( h \) the flow depth (Johnson, 1970; Johnson & Rodine, 1984; Sohn, 2000; Wang et alii, 2000). Due to this, each surge retains entirety and the superposition of successive surges make up a deposition. In a wide open slope one can distinguish different surges by the bifurcations of the distal ends, the lateral margins (see Fig. 2), and sometimes the overlapping wedges (e.g. Sohn, 2000).

**DISTRIBUTION OF FLOW DEPTH**

**THEORETIC DERIVATION**

As debris flow deposits by superposition of many surges in a random way, we apply a probabilistic viewpoint. At first we consider the distribution of the flow velocity, which is the most dynamic parameter of the surge. We find that the velocity satisfies the Weibull distribution, which in the form of exceedance probability is:

\[
P (v) = \exp (- (v/a)^b)
\]

\[\text{Tab. 1 - Parameters of the Weibull distribution for debris flow velocity} \]

<table>
<thead>
<tr>
<th>Events</th>
<th>Surge number</th>
<th>( a )</th>
<th>( b )</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>870627</td>
<td>51</td>
<td>5.3086</td>
<td>4.3165</td>
<td>The standard error of the parameters range between 0.06 and 0.36; the estimated covariance of the parameter estimates is ( 10^{-3} ) in order.</td>
</tr>
<tr>
<td>890627</td>
<td>120</td>
<td>7.2526</td>
<td>3.7949</td>
<td></td>
</tr>
<tr>
<td>890802</td>
<td>129</td>
<td>6.4976</td>
<td>4.2065</td>
<td></td>
</tr>
<tr>
<td>890803</td>
<td>166</td>
<td>3.4521</td>
<td>4.4070</td>
<td></td>
</tr>
<tr>
<td>900718</td>
<td>125</td>
<td>6.4299</td>
<td>3.1536</td>
<td></td>
</tr>
<tr>
<td>900729</td>
<td>130</td>
<td>6.2702</td>
<td>4.8055</td>
<td></td>
</tr>
<tr>
<td>910708</td>
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<td>4.9363</td>
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<td></td>
</tr>
<tr>
<td>910709</td>
<td>427</td>
<td>6.1992</td>
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<td></td>
</tr>
<tr>
<td>910711</td>
<td>253</td>
<td>6.6009</td>
<td>4.3055</td>
<td></td>
</tr>
<tr>
<td>910715</td>
<td>114</td>
<td>5.9775</td>
<td>3.8492</td>
<td></td>
</tr>
<tr>
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<td>6.5187</td>
<td>3.4169</td>
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</tr>
<tr>
<td>910813</td>
<td>348</td>
<td>6.6348</td>
<td>4.9787</td>
<td></td>
</tr>
<tr>
<td>920721</td>
<td>79</td>
<td>6.2320</td>
<td>3.8371</td>
<td></td>
</tr>
<tr>
<td>930826</td>
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<td>5.9408</td>
<td>3.5212</td>
<td></td>
</tr>
<tr>
<td>940616</td>
<td>151</td>
<td>7.6137</td>
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<td></td>
</tr>
<tr>
<td>940702</td>
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</tr>
<tr>
<td>950715</td>
<td>265</td>
<td>8.7384</td>
<td>4.1042</td>
<td></td>
</tr>
</tbody>
</table>

where \( P (v) \) is the probability of surge with velocity bigger than \( v \). Statistics for some events are listed in Table 1.

As for the flow depth, it is found to be related to velocity by a power law (Li et alii, 2005; 2010):

\[
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\[
P (h) = C \exp (- \frac{h}{a_H}^b)
\]

\[\text{Fig. 1 - Deposition of individual surges} \]

\[\text{Fig. 3 displays the data points of three events, with the fitting parameters (}\ a_H, b_H\text{ listed in the legend box. The calculated results for some events are listed in Tab. 2 and Fig. 8 presents the data in log-log plot.} \]
over surges of each event. This leads to the coincidence of data points from different events (Fig. 5).

Therefore, all the events are subject to the same

<table>
<thead>
<tr>
<th>Events</th>
<th>Surge number</th>
<th>$c$</th>
<th>$a_{11}$</th>
<th>$b_{11}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>870027</td>
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<td>1.015</td>
<td>0.5345</td>
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</tr>
<tr>
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<td>1.066</td>
<td>1.102</td>
<td>1.614</td>
<td>0.9913</td>
</tr>
<tr>
<td>890802</td>
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<td>1.024</td>
<td>5.4930</td>
<td>2.728</td>
<td>0.9279</td>
</tr>
<tr>
<td>890803</td>
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<td>0.9626</td>
<td>4.2710</td>
<td>2.449</td>
<td>0.9691</td>
</tr>
<tr>
<td>900718</td>
<td>125</td>
<td>1.068</td>
<td>0.9662</td>
<td>1.633</td>
<td>0.9971</td>
</tr>
<tr>
<td>900729</td>
<td>130</td>
<td>1.001</td>
<td>1.085</td>
<td>1.951</td>
<td>0.9970</td>
</tr>
<tr>
<td>910708</td>
<td>201</td>
<td>1.060</td>
<td>1.044</td>
<td>1.576</td>
<td>0.9954</td>
</tr>
<tr>
<td>910709</td>
<td>427</td>
<td>1.109</td>
<td>1.514</td>
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</tr>
<tr>
<td>910711</td>
<td>253</td>
<td>1.010</td>
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<td>1.502</td>
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<tr>
<td>910715</td>
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<tr>
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<td>910813</td>
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<td>1.003</td>
<td>0.771</td>
<td>2.175</td>
<td>0.9999</td>
</tr>
<tr>
<td>920721</td>
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</tr>
<tr>
<td>930826</td>
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<td>1.063</td>
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<td>0.9568</td>
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<td>1.753</td>
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<tr>
<td>950715</td>
<td>265</td>
<td>0.9769</td>
<td>0.5202</td>
<td>1.509</td>
<td>0.9895</td>
</tr>
</tbody>
</table>

It is noted that the coefficient $C$ is exclusively near 1 (the average is about 1.04 and the standard variance is 0.009), and the fitting curve is fine with $R^2$ near 1. This confirms the validity of the Weibull distribution. Besides, the shape parameter $b_{H}$ varies little, with average 1.85 and variance 0.21.

The scale parameter $a_{H}$ also varies little with several exceptions. $a_{H}$ for events 890802 and 890803 are high (5.49 and 4.21) and for events 870627 and 950715 are small (0.53 and 0.52). These abnormalities correspond to the fact that event with big $a_{H}$ is composed of “shallow” surges in depth of less than 1.0m and that event with small $a_{H}$ is composed of “deep” surges having relatively high flow depth. As shown in Fig. 4, the abnormal events are “off” the central data points.

More importantly, the abnormality of scale parameter can be eliminated by rescaling the depth by $h^* = h/(\bar{h}/\bar{h})$, where $\bar{h}$ denotes average
distribution on average level, having almost the same parameters. The average value of $a_H$ and $b_H$ are 1.04 and 1.75, with variance of 0.087 and 0.079, respectively. This agrees well with the rough estimate of Eq.(5). And for individual events, even better agreement can be achieved. Consider the event 910715, for example, $k = 5.97$, $n = 0.42$; and the Weibull parameter for velocity is $a = 5.98$ and $b = 3.85$ (Tab. 1). Then one gets $a_H = 0.99$ and $b_H = 1.61$. Thus the distribution parameters are well fixed and can be easily estimated to a satisfactory accuracy from the velocity probability. This is a very admirable virtue for practice in risk assessment.

**PARAMETER DETERMINATION**

In order to use the distribution for risk assessment, we should determine the parameters in general. Following discussions above, we assume that the shape parameter is universally applicable and the scale parameter is related to the average value. According to Weibull distribution, the average velocity $\langle v \rangle$ is

$$\langle v \rangle = a \Gamma(1 + 1/b)$$

where $\Gamma(1 + x) = x\Gamma(x) \ (x > 0)$ is the Gamma function. For the case of JJG, $1/b$ is less than 1/3 (Tab. 1), then $\langle v \rangle = a/b\Gamma(1/b) > 0.9a$. Thus the scale parameter $a$ can be estimated by the average velocity at the accuracy of 90%. For a valley to be assessed, we suppose the expected velocity is $\beta$ times the maximal velocity of JJG, then the same factor $\beta$ also applies to the average value of the expected velocity. Thus the scale parameter for velocity distribution is $\beta a$.

The parameters for flow depth distribution can be similarly derived. Corresponding to the velocity, the average depth is

$$\langle H \rangle = \beta a/k\Gamma(1 + 1/bn) = \beta a/(kbn)\Gamma(1/bn) \sim \beta a/k$$

(7)

Again, the value $bn = 1.60$ is used here to estimate the average value. Therefore the distribution of flow depth can be well derived from the designed velocity.

**IMPLICATIONS FOR RISK ASSESSMENT**

The distribution derived above provides an easy assessment of risk. Specifically, the depth distribution can be used to evaluate the inundated area of a potential debris flow. The inundated area is hard to determine in practice because of the complexity of landform and the uncertainty of the flow. Instead, it is usually estimated by a postulated discharge or the designed discharge for engineering structure.

In engineering design, a discharge ($Q_p$) is usually presumed by a given frequency corresponding to the rainfall and then the discharge is used to determine the cross-section and velocity. This methodology ignores any variations of debris flow which might be considerable even under the given condition.

On the other hand, discussions above suggest that debris flow occurs in a random way and the velocity conforms to a certain probability distribution. Besides, the distribution parameters vary slightly with events; it is possible to suppose the distribution holds in general. This is reasonable because the velocity is mainly determined by the fluid physics and the flow regimes of debris flow are similar in various conditions. Reports of debris flow in other areas also indicate that velocity varies in the similar range, mainly between 5 m/s and 15 m/s. This means the distribution is generally applicable for assessment. Consider the distribution with average parameters, i.e., $a = 6.4$, $b = 4.1$, and $t = a^b = 0.0005$ (table 1):

$$P(v) = \exp(-t v^b) = \exp(-0.0005 v^{4.1})$$

(8)

The medium velocity (i.e. with probability of 50%) is $v = 5.84$ (m/s), and the probability of $v > 9.26$ is less than 1%. The 1%-possible velocity can be properly taken as the maximum of velocity in general cases. Correspondingly, the maximal flow depth of 1% possibility satisfies $P(H) = \exp(-a_H H^{bn}) \sim 0.01$. As $a_H \sim 1.0$ and $b_H \sim 1.6$ on average, this gives $H \sim 2.6m$.

Then we can derive the corresponding discharge $Q_{as} = VH_{S}$

(9)

where $S$ is the wet perimeter of the cross-section passed by the flow and can be measured in field, and $H_{S}$ gives the area of cross-section. For example, Fig. 6 shows a typical cross-section in a debris-flow channel which retains surge marks of different flow depth.
1) Debris flow moves in manner of separate surges and the surge acts as the unit of flow motion and deposition. This is determined by the nature of the fluid but not by the environmental factors;

2) Debris flows in JJJG originate from various sources and under different rainfalls; therefore they represent a variety of physical conditions. Moreover, the surges cover a wide spectrum of motion regimes. In other words, debris flow in JJJG presents properties and performances of debris flows in various regions and conditions;

3) The distribution parameters fall into a small range; this means that individual events conform to the same rule despite their varieties of origins.

Therefore the probability distribution is expected to be generally applicable for debris flow in various regions and conditions, only with small variation of scale parameters that doesn’t change the form of distribution.

Discharge estimated through the distribution is expected to be more reliable because it incorporates the living performance of debris flow surge other than derives from the indirect conditions of debris flow, such as the background and the rainfall.

Additionally, although there are rare valleys like JJJG that develop debris flows with such a high frequency and variety, debris flow is probabilistic even if only one event falls in a valley. Thus the probabilistic scenario we get from JJJG might as well provide a prototype for assessing debris flows in different valleys.

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