SIMULATION BASED ON FINITE VOLUME METHOD OF THE ENTRAIMENT OF DEBRIS FLOW

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ABSTRACT

The mountain hazards like snow avalanches, landslides, rock falls, debris flows all have strong destructive power which seriously threaten human lives and belongings. Therefore, it is necessary to study more about the happenings and developments of these disasters. Among the important and common features of the debris flow, the entrainment is the one that can increase the volume of debris flow and affects significantly the flow motion. These influences usually result in a more harmful and stronger destructive power. With the method of finite volume discretization, the numerical simulation of the entrainment process of debris flow is achieved in this paper. The influences of entrainment capabilities on the motion and deposition of debris flow are mainly studied. The typical engineering examples show and prove the stability and effectivity of numerical methods.

Key words: debris flow, entrainment, finite volume, dynamic, simulation

INTRODUCTION

The dynamic simulation of debris flow can not only improve human cognition in the regularity and characteristics of disasters by inversing and reappearing the development law of debris flow disasters, but also predict the disaster-caused area and intensity of debris flow, and consequently, scientific references of industry planning and construction, and the measures of prevention and reduction of disasters can be offered. Therefore, the dynamic simulation of debris flow is significant and valuable.

In the development of debris flow, the surface bulk material along the way would have a shear failure and slide with debris flow during the motion process so that the entrainment is formed which increase the volume of debris flow and change the composition of motion material. For this reason, the surface material is an important factor in the development of debris flow and should be considered in its dynamic simulation. At the moment, many scholars have begun their studies on the entrainment problems during the motion of debris flow and some progress has been made.

In order to realize the dynamic simulation of debris flow entrainment and study the influences of this entrainment on dynamic simulation, the back analysis is made to study a typical historical debris flow, Nomash River in Canada, by combining entrainment dynamic model theory which considers the entrainment effects and the method of finite volume discretization based on the approximate Riemann solver of HLL Scheme. The calculation also takes the variation of terrain slope caused by entrainment into account. The calculation results which are identical to actual disaster-caused area confirm the effectivity of the dynamic model theory and the numerical solution. Compared with the results which have no consideration of entrainment, the influence of entrainment action on dynamic process of debris flow is analysed.
**DYNAMIC MODEL THEORY**

**BASIC EQUATION**

Assuming that the density of debris flow is invariable during the motion process and ignoring the variation of physical variable in the direction of flow depth, the dynamic model equation is as follows:

\[
\frac{\partial U}{\partial t} + \frac{\partial E(U)}{\partial x} + \frac{\partial H(U)}{\partial y} = S
\]

(1)

Where \( U \) is conservation variable, \( F(E,H) \) is calculation flux, \( S \) is source item, these items can be expressed as follows:

\[
E = \left( \frac{hu^2 + k_{xy}gh^2}{2} \right), \quad H = \left( \frac{hv}{l_{uv}} \right)
\]

\[
U = \left( \begin{array}{c}
h \\ hu \\
hv \\
\end{array} \right), \quad S = \left( \begin{array}{c}
E_x \\ -gh\zeta_x / \partial x + S_p \\
-gh\zeta_y / \partial y + S_p \\
\end{array} \right)
\]

(2)

In this expression, \( H \) is the depth of flow, \( u \) and \( v \) are respectively the velocities in \( x \) direction and \( y \) direction, \( g \) is acceleration of gravity, \( E \) is the source item of entrainment velocity, \( \zeta \) is elevation on bottom surface in the coordinate system, and \( S_p \) is the source item of resistance, they can be used according to different conditions, \( k_{xy} \) is the coefficient of lateral stress which can be expressed as (Savage & Hutter, 1989; Iverson & Denlinger, 2001; Pirulli et alii, 2007):

\[
k_{xy} = \frac{2}{\cos^3 \phi} \left( \frac{1}{\cos^2 \phi} - 1 \right)
\]

(3)

where \( \phi \) is internal friction angle and \( \delta \) is bottom friction angle. Different motion states correspond with different coefficients of lateral stress. When \( \partial u / \partial x + \partial v / \partial y > 0 \), the motion is in active state, however when \( \partial u / \partial x + \partial v / \partial y < 0 \), the motion is in passive state (Savage & Hutter, 1989; Iverson & Denlinger, 2001; Pirulli et alii, 2007).

**RESISTANCE MODEL**

Choosing a correct resistance model of the model equation of dynamic theory is an important step in the dynamic simulation of debris flow. In this paper, analysis is achieved by use of friction resistance and Voellmy resistance.

(1) Friction resistance

During the process of debris flow, the resistance is mainly from the bottom friction, and the friction resistance can be considered as (Savage & Hutter, 1989; Iverson & Denlinger, 2001; McDougall & Hung, 2005; Pirulli et alii, 2007; Hung, 2009):

\[
S_p = -\frac{u}{|v|} \rho g h \tan \delta, \quad S_h = \frac{v}{|v|} \rho g h \tan \delta
\]

(4)

where \( v=(u,v) \) is velocity vector.

(2) Voellmy resistance

If there exists water during the motion process of debris flow, its resistance effect should also be considered, in this case, Voellmy resistance is usually used as the resistance model (Hung & Evans, 2004; McDougall & Hung, 2005; Hung, 2009):

\[
S_p = -\frac{u}{|v|} f \rho g h \text{sgn}(u) \frac{\delta \text{sgn}(v)}{\xi}
\]

\[
S_h = -\frac{v}{|v|} f \rho g h \text{sgn}(v) \frac{\delta \text{sgn}(u)}{\xi}
\]

(5)

where \( \text{sgn}(\alpha) \) is sign function which can be described as:

\[
\text{sgn}(\alpha) = \begin{cases} 1 & a > 0 \\ 0 & a = 0 \\ -1 & a < 0 \end{cases}
\]

(6)

In eq. (5), \( f \) is friction coefficient, \( \xi \) is hydraulic parameter, the first item from the right hand represents friction resistance which has the same form with eq. (4), \( f \) corresponds to \( \tan \delta \), and the second item represents other resistances, which are experientially used to express the items relative to velocity in the analysis.

**ENTRAINMENT VELOCITY**

The entrainment velocity form of Hung is used in this paper (Hung & Evans, 2004; McDougall & Hung, 2005), so the relationship between entrainment velocity \( E \) and rate of increase of entrainment \( E_s \) is as follows:

\[
E_s = E_b h |v|
\]

(7)

A reasonable \( E_s \) can be obtained only if it is adjusted according to the actual condition repeatedly. One feasible method is to use the average rate of increase \( E_s \), obtained by use of the total volume before and after entrainment, which can be describes as:

\[
\bar{E}_s = \frac{\ln(V_f/V_0)}{d}
\]

(8)

where \( V_0 \) is the total volume before entrainment, and \( V_f \) after entrainment, and \( d \) is the approximate average length in entrainment area.

**FINITE VOLUME NUMERICAL METHOD**

**FINITE VOLUME DISCRETIZATION**

The computational domain is discretized by using the layout form of the triangle unstructured triangular meshes and Cell Center which have a great adaptability.
Changing eq. (1) into vector form, then find the integral of this vector form:
\[
\frac{\partial U}{\partial t} + \int_{V_i} \nabla \cdot F dv = \int_{\partial V_i} S d\Gamma
\]  
(9)

Considering the \( U_i \) and \( S_i \) as the average values of element variable and source item respectively, and placing them in the centre of element, then:
\[
U_i = \frac{1}{A_i} \int_{V_i} U dV, \quad S_i = \frac{1}{A_i} \int_{V_i} S d\Gamma
\]  
(10)

By using the Green formula to transform eq. (9) from surface integral to line integral, it is obtained that:
\[
\frac{\Delta U_i}{\Delta t} + \int_{\partial A_i} F \cdot n dl = S_i A_i
\]  
(11)

where \( L \) is the boundary of the \( i \)th element, \( A_i \) is the surface of element \( i \), \( n=(n_x,n_y)=(\cos \theta, \sin \theta) \) is the direction of exterior normal, and \( \theta \) is the angle between the exterior normal and \( x \)-axis.

The following equation can be obtained by discretizing eq. (11):
\[
\Delta U_i = \frac{\Delta t}{A_i} \sum_{j=1}^{N} (F_{ij}) \Delta y_j + \Delta S_i
\]  
(12)

where \( \Delta U_i \) is the increment of variable, \( \Delta y_j \) is the side length of element, \( F_{ij} \) represents the normal numerical flux at the \( j \)th edge of the \( i \)th element.

**NUMERICAL FLUX CALCULATION**

The present model employs the HLL Riemann solver to compute the normal flux at the face of an element as follows (Harten et alii, 1983; Aliparast, 2009):
\[
F^* \cdot n = \begin{cases} 
F_L \cdot n & s_L \geq 0 \\
F_L \cdot n - s_L (F_L - F_R) & 0 < s_R < s_L \\
F_R \cdot n & s_R \leq 0
\end{cases}
\]  
(13)

where \( L \) and \( R \) are variables at the inner edge and outer edge respectively; \( (U_L)_{ij} \) and \( (U_R)_{ij} \) are the \( U \) value on the left and right sides at the \( j \)th edge of the \( i \)th element, respectively; \( s_L \) and \( s_R \) are wave velocities of the both edges, respectively, it can be expressed that:
\[
s_L = \min(\mu U_L + v_L n_y - c_L, u^* - c^* - c) \\
s_R = \max(\mu U_R + v_R n_y + c_R, u^* + c^*)
\]  
(14)

where \( u^* = \frac{1}{2} (U_L + U_R) n_x + (v_L + v_R) n_y + c_L - c_R \) and \( c^* = \frac{c_L + c_R}{2} + \frac{(U_L - U_R) n_x + (v_L - v_R) n_y}{4} \)

when the left side is dry, \( s_L \) and \( s_R \) can be calculated as follows:
\[
s_L = u_L n_x + v_L n_y - 2c_R \quad (h_L = 0)
\]  
(15)

when the right side is dry, \( s_L \) and \( s_R \) can be calculated as follows:
\[
s_L = u_L n_x + v_L n_y - c_L \quad (h_R = 0)
\]  
(16)

**SPATIAL SECOND-ORDER LINEAR RECONSTRUCTION**

When the numerical discretization method of finite volume is employed, the variable of the control volume of spatial second-order accuracy should be distributed linearly. With the MUSCL method (Osher, 1996), the states of the both edges are concluded by interpolating variables, the linear reconstruction on unstructured mesh is as described in Fig.1.

In Fig. 1, \( U_i, V_i, U_j, V_j, U_s, V_s \) and \( U_i \) are centroid values of the element, \( U^L_i, U^R_i, U^S_i, V^L_i, V^R_i, V^S_i \) and \( U_s \) are linear interpolation, while \( U^L_j \) and \( U^R_j \) are the linear upwind values at the both edges, obtained by extrapolating variables, \( U^C_{ij} \) is the central value at the edge.

The limited linear interpolation values are used to calculated the variables at the both edges:
\[
(U_i)_j = U_i + \Phi(c^*_0)(U^C_i - U^L_i) \\
(U_s)_j = U_i + \Phi(c^*_0)(U^C_i - U^R_i)
\]  
(17)

where the slope of linear interpolation can be expressed as:
\[
\Phi(r) = \frac{r + |r|}{1 + \max(1, |r|)}
\]  
(18)

Besides, with Runge-Kutta methods (Hubbard et alii, 1999), the format of space-time second order accuracy is obtained.
CALCULATION OF TERRAIN SLOPE

Among the source items, the derivative term of elevation on bottom surface should be especially treated. On unstructured triangular meshes, through the calculation with the coordinate values of the three nodes of the triangle element, this derivative term can be obtained as follows:

\[
\frac{\partial z}{\partial x} = \frac{1}{2A}[(z_{i0}(x_1-x_i)+z_{i1}(x_1-x_i)+z_{i2}(x_1-x_i))]
\]
\[
\frac{\partial z}{\partial y} = \frac{1}{2A}[(z_{i0}(y_1-y_i)+z_{i1}(y_1-y_i)+z_{i2}(y_1-y_i))]
\]

(21)

where \((x_i, y_i)\) represents the coordinate value of the \(k\) th node of the \(j\) th triangle, \(z_{ik}\) represents the elevation on bottom surface of the \(k\) th node in the coordinate system.

During the process of entrainment, elevation on bottom surface changes with the reduction of the bottom mass, at this time, the elevation on bottom surface reconstructs linearly in space like other variables above-mentioned, so the derivative term of the source items can be calculated with the reconstructed values of the element:

\[
\frac{\partial z}{\partial x} = \frac{1}{A}[(z_{i0}(x_1-x_i)+z_{i1}(x_1-x_i)+z_{i2}(x_1-x_i))]
\]
\[
\frac{\partial z}{\partial y} = \frac{1}{A}[(z_{i0}(y_1-y_i)+z_{i1}(y_1-y_i)+z_{i2}(y_1-y_i))]
\]

(22)

where \((x', y')\) represents the coordinate value of the middle point of the \(j\) th edge of the \(i\) th triangle element, \(z_{ij}\) represents the internal elevation of bottom surface of the \(j\) th edge which is obtained through the spatial second-order linear reconstruction.

CALCULATION AND ANALYSIS OF AN EXAMPLE

NOMASH RIVER DEBRIS FLOW

Nomash River lies in the west part of British Columbia, Canada, the area bedrock is mainly made of crystalline limestone.

In April 1999, a debris flow occurred in the upper course of the Nomash River during spring snowmelt, as shown in Fig. 2. The source area located on a mountain slope angled at 50° to the northeast of the U-shaped river valley. At the beginning, a continuous shear plane was formed after a series of crossed joints about 430 m above the river, then 300 000 m³ of crystalline limestone was separated and suddenly collapsed. Assuming 25% bulking of the source failure, there was 375.000 m³ of fragments produced by the slide, which fell into the valley of the Nomash River perpendicularly (HUNGR & EVANS, 2004; MCDougALL & HUNGR, 2005).

Before reaching the valley bottom, an additional 360 000 m³ of saturated colluvium was impacte by the entrainment, and the entrainment ration reached 0.96 which is really alarming number. After the entrainment, a 100–150 m-wide erosion area was left, the vestige and trace of entrainment are visible in Fig. 3.

FIG. 2 - Nomash River debris flow

FIG. 3 - Entrainment area of Nomash River debris flow
is very significant in the dynamic process of debris flow. The final results in Fig.5 obtained at the 40th second are calculated without considering entrainment effects.

It can be seen from Fig.4 that without regard to entrainment, the deposit will stop at the valley bottom because of lacking forces which can push it to climb the slope, as a result, the disaster-caused area is much smaller than that is shown in Fig.4. Thus it is known that entrainment plays a very important role during a dynamic process of debris flow. It makes debris flow more powerful and harmful and should be considered as a key factor in the formation of large-scale debris flows.

It can be seen from the calculation that the entrainment effect which greatly increases the motion volume resistance model is used, and the calculation parameters are $\xi=400 \text{ m/s}^2$ and $f=0.05$. With the above method, the results at different time are obtained as shown in Fig.4.

It can be seen from Fig.4 that the thick real line represents the real disaster-caused area, the interval between the geography contours is 20m, and the flow depth contour is 1m. It can be seen from the above calculation that the back analysis shows the whole motion process of the debris flow: at the 30th s, the flow motion after the entrainment almost arrived at the highest point of the opposite mountain slope, at the 60th s, the flow motion went backward after a second turning, and the deposit was gradually formed. Certes, there still exists some differences between the calculation results and actual situation, and the calculation results could be more exact if a more reasonable modification of geography can be made and the calculation parameters of entrainment of path material which can better conform to the actual situation are applicable. Nevertheless, it is still satisfactory that the simulation results are relatively identical to actual disaster-caused area.

It can be seen from the calculation that the entrainment effect which greatly increases the motion volume...
CONCLUSIONS

Entrainment is one of the most important factors that can not be ignored in the dynamic process of debris flow, which should be considered in the dynamic simulation of debris flows.

The dynamic simulation of the entrainment of debris flow is realized by combining entrainment dynamic model theory and the method of finite volume discretization. The variation of terrain slope caused by entrainment is also considered in the simulation. The back analysis is made to study Nomash River debris flow, a typical historical debris flow happened in Canada. The calculation results which are relatively identical to actual disaster-caused area confirm the effectiveness of the dynamic model theory and the numerical solution. Compared with the results which have no consideration of entrainment, it is can be seen that the entrainment action can increase the motion volume of entrainment, change the composition of motion materials and enhance the motility of debris flow, so that the entrainment will be more destructive and harmful. In fact, entrainment is a main cause of large-scale debris flow, so it should be paid more attention in the dynamic simulation of debris flow.

REFERENCES


